Mark Scheme (Results)

January 2022

Pearson Edexcel International A Level
In Further Pure Mathematics F3 (WFM03)
Paper 01

Question Number	Scheme	Notes	Marks
1(a)	$$8\cosh^4 x = 8\left(\frac{e^x + e^{-x}}{2}\right)^4 = \frac{8}{16}\left(e^{4x} + 4e^{2x} + 6 + 4e^{-2x} + e^{-4x}\right)$$ Applies $\cosh x = \dfrac{e^x + e^{-x}}{2}$ and attempts to expand the bracket to at least 4 different and no more than 5 different terms of the correct form but they may be "uncollected" depending on how they do the expansion. Allow unsimplified terms e.g. $\left(e^x\right)^3 e^{-x}$. May see $8\left(\dfrac{e^x + e^{-x}}{2}\right)^2\left(\dfrac{e^x + e^{-x}}{2}\right)^2$ but must attempt to expand as above		M1
	$$= \frac{1}{2}\left(e^{4x} + e^{-4x}\right) + 4\left(\frac{e^{2x} + e^{-2x}}{2}\right) + 3 = \dots$$	Collects appropriate terms and reaches the form $\cosh 4x + p\cosh 2x + q$ or obtains values of p and q.	M1
	$$= \cosh 4x + 4\cosh 2x + 3$$	Correct expression or values e.g. $p = 4$ and $q = 3$	A1
	No marks are available in (a) if exponentials are not used but note that they may appear in combination with the use of hyperbolic identities e.g.: $$8\cosh^4 x = 8\left(\cosh^2 x\right)^2 = 8\left(\frac{\cosh 2x + 1}{2}\right)^2 = 2\left(\frac{e^{2x} + e^{-2x}}{2} + 1\right)^2$$ $$= 2\left(\frac{e^{4x} + 2 + e^{-4x}}{4} + e^{2x} + e^{-2x} + 1\right) = \frac{e^{4x} + e^{-4x}}{2} + 4\left(\frac{e^{2x} + e^{-2x}}{2}\right) + 2$$ $$= \cosh 4x + 4\cosh 2x + 3$$ **Allow to "meet in the middle" e.g. expands as above and compares with** $$\frac{1}{2}\left(e^{4x} + e^{-4x}\right) + p\left(\frac{e^{2x} + e^{-2x}}{2}\right) + q \Rightarrow p = \dots, q = \dots$$ **but to score any marks the expansion must be attempted.**		
			(3)

(b) Way 1	$\cosh 4x - 17\cosh 2x + 9 = 0 \Rightarrow 8\cosh^4 x - 4\cosh 2x - 3 - 17\cosh 2x + 9 = 0$ $\Rightarrow 8\cosh^4 x - 21\cosh 2x + 6 = 0 \Rightarrow 8\cosh^4 x - 21\left(2\cosh^2 x - 1\right) + 6 = 0$ Uses **their** result from part (a) and $\cosh 2x = \pm 2\cosh^2 x \pm 1$ to obtain a quadratic equation in $\cosh^2 x$ **or** $\cosh 4x - 17\cosh 2x + 9 = 0 \Rightarrow 2\left(2\cosh^2 x - 1\right)^2 - 1 - 17\left(2\cosh^2 x - 1\right) + 9 = 0$ Uses $\cosh 4x = \pm 2\cosh^2 2x \pm 1$ and $\cosh 2x = \pm 2\cosh^2 x \pm 1$ to obtain a quadratic equation in $\cosh^2 x$		M1
	$\Rightarrow 8\cosh^4 x - 42\cosh^2 x + 27 = 0$	Correct 3TQ in $\cosh^2 x$	A1
	$\Rightarrow 8\cosh^4 x - 42\cosh^2 x + 27 = 0$ $\Rightarrow \cosh^2 x = \dfrac{9}{2}\left(,\dfrac{3}{4}\right)$	Solves 3TQ in $\cosh^2 x$ (apply usual rules if necessary) to obtain $\cosh^2 x = k$ $\left(k \in \mathbb{R} \text{ and } > 1\right)$. May be implied by their values – check if necessary.	M1
	$\cosh^2 x = \dfrac{9}{2} \Rightarrow \cosh x = \dfrac{3}{\sqrt{2}} \Rightarrow x = \pm\ln\left(\dfrac{3}{\sqrt{2}} + \sqrt{\dfrac{9}{2} - 1}\right)$ or $\cosh x = \dfrac{3}{\sqrt{2}} \Rightarrow \dfrac{e^x + e^{-x}}{2} = \dfrac{3}{\sqrt{2}} \Rightarrow \sqrt{2}e^{2x} - 6e^x + \sqrt{2} = 0 \Rightarrow e^x = ... \Rightarrow x = ...$ or $\cosh^2 x = \dfrac{9}{2} \Rightarrow \left(\dfrac{e^x + e^{-x}}{2}\right)^2 = \dfrac{9}{2} \Rightarrow e^{4x} - 16e^{2x} + 1 = 0 \Rightarrow e^{2x} = ... \Rightarrow x = ...$ Takes square root to obtain $\cosh x = k$ $\left(k > 1\right)$ and applies the correct logarithmic form for arcosh or uses the correct exponential form for $\cosh x$ to obtain at least one value for x The root(s) must be real to score this mark.		M1
	$x = \pm\ln\left(\dfrac{3\sqrt{2}}{2} + \dfrac{\sqrt{14}}{2}\right)$ Both correct and exact including brackets. Accept simplified equivalents e.g. $x = \ln\left(\dfrac{3}{\sqrt{2}} \pm \dfrac{\sqrt{7}}{\sqrt{2}}\right)$ but withhold this mark if additional answers are given unless they are the same e.g. allow $x = \pm\ln\left(\dfrac{3\sqrt{2}}{2} \pm \dfrac{\sqrt{14}}{2}\right)$		A1
			(5)

(b) Way 2	$\cosh 4x - 17\cosh 2x + 9 = 0 \Rightarrow 2\cosh^2 2x - 1 - 17\cosh 2x + 9 = 0$ Applies $\cosh 4x = \pm 2\cosh^2 2x \pm 1$ to obtain a quadratic equation in $\cosh 2x$		M1
	$2\cosh^2 2x - 17\cosh 2x + 8 = 0$	Correct 3TQ in $\cosh 2x$	A1
	$2\cosh^2 2x - 17\cosh 2x + 8 = 0$ $\Rightarrow \cosh 2x = 8\left(, \dfrac{1}{2}\right)$	Solves 3TQ in $\cosh 2x$ (apply usual rules if necessary) to obtain $\cosh 2x = k \ (k \in \mathbb{R} \ \text{and} > 1)$	M1
	$\cosh 2x = 8 \Rightarrow 2x = \pm\ln\left(8 + \sqrt{8^2 - 1}\right)$ or $\cosh 2x = 8 \Rightarrow \dfrac{e^{2x} + e^{-2x}}{2} = 8 \Rightarrow e^{4x} - 16e^{2x} + 1 = 0 \Rightarrow e^{2x} = ... \Rightarrow 2x = ...$ Applies the correct logarithmic form for arcosh from $\cosh 2x = k \ (k > 1)$ or uses the correct exponential form for $\cosh 2x$ to obtain at least one value for $2x$ The root(s) must be real to score this mark.		M1
	$x = \pm\dfrac{1}{2}\ln\left(8 + 3\sqrt{7}\right)$ or e.g. $x = \pm\ln\left(8 + 3\sqrt{7}\right)^{\frac{1}{2}}$	Both correct and exact with brackets. Accept simplified equivalents e.g. $x = \dfrac{1}{2}\ln\left(8 \pm \sqrt{63}\right)$ but withhold this mark if additional answers are given unless they are the same as above.	A1
(b) Way 3	$\cosh 4x - 17\cosh 2x + 9 = 0 \Rightarrow \dfrac{e^{4x} + e^{-4x}}{2} - \dfrac{17}{2}\left(e^{2x} + e^{-2x}\right) + 9 = 0$ $\Rightarrow e^{8x} - 17e^{6x} + 18e^{4x} - 17e^{2x} + 1 = 0$ M1: Applies the correct exponential forms and attempts a quartic equation in e^{2x} A1: Correct equation		M1A1
	$e^{8x} - 17e^{6x} + 18e^{4x} - 17e^{2x} + 1 = 0$ $\Rightarrow e^{2x} = 8 \pm 3\sqrt{7}, ...$	Solves and proceeds to a value for e^{2x} where $e^{2x} > 1$ and real.	M1
	$\Rightarrow e^{2x} = 8 \pm 3\sqrt{7} \Rightarrow 2x = \ln\left(8 \pm 3\sqrt{7}\right)$	Takes ln's to obtain at least one value for $2x$ The root(s) must be real to score this mark.	M1
	$x = \dfrac{1}{2}\ln\left(8 \pm 3\sqrt{7}\right)$ or e.g. $x = \ln\left(8 \pm 3\sqrt{7}\right)^{\frac{1}{2}}$	Both correct and exact with brackets. Accept simplified equivalents e.g. $x = \pm\dfrac{1}{2}\ln\left(8 + 3\sqrt{7}\right)$ but withhold this mark if additional answers are given unless they are the same as above.	A1
			Total 8

Question Number	Scheme	Notes	Marks
2	$$\dfrac{\mathrm{d}x}{\mathrm{d}\theta} = \dfrac{\sec\theta\tan\theta + \sec^2\theta}{\sec\theta + \tan\theta} - \cos\theta$$ Correct derivative. Do not condone missing brackets e.g. $\dfrac{\mathrm{d}x}{\mathrm{d}\theta} = \dfrac{1}{\sec\theta + \tan\theta} \times \sec\theta\tan\theta + \sec^2\theta - \cos\theta$ unless a correct expression is implied by subsequent work. Award when a correct expression is seen but note that other forms are possible e.g. $\sec\theta - \cos\theta, \ \tan\theta\sin\theta$		B1
	$$\left(\dfrac{\mathrm{d}x}{\mathrm{d}\theta}\right)^2 + \left(\dfrac{\mathrm{d}y}{\mathrm{d}\theta}\right)^2 = \left(\dfrac{\sec\theta\tan\theta + \sec^2\theta}{\sec\theta + \tan\theta} - \cos\theta\right)^2 + \left(-\sin\theta\right)^2$$ Attempts $\dfrac{\mathrm{d}y}{\mathrm{d}\theta}$ and then $\left(\dfrac{\mathrm{d}x}{\mathrm{d}\theta}\right)^2 + \left(\dfrac{\mathrm{d}y}{\mathrm{d}\theta}\right)^2$		M1
	$$S = (2\pi)\int \cos\theta\sqrt{\left(\dfrac{\mathrm{d}x}{\mathrm{d}\theta}\right)^2 + \left(\dfrac{\mathrm{d}y}{\mathrm{d}\theta}\right)^2}\,\mathrm{d}\theta$$ $$= (2\pi)\int \cos\theta\sqrt{\left(\dfrac{\sec\theta\tan\theta + \sec^2\theta}{\sec\theta + \tan\theta} - \cos\theta\right)^2 + \left(-\sin\theta\right)^2}\,\mathrm{d}\theta$$ Applies a correct surface area formula using their $\dfrac{\mathrm{d}x}{\mathrm{d}\theta}$ and their $\dfrac{\mathrm{d}y}{\mathrm{d}\theta}$ with or without the 2π For reference: $\sqrt{\left(\dfrac{\sec\theta\tan\theta + \sec^2\theta}{\sec\theta + \tan\theta} - \cos\theta\right)^2 + \left(-\sin\theta\right)^2} = \tan\theta$ Allow π in front of the integral but must be an integral		M1
	$$(2\pi)\int \sin\theta\,\mathrm{d}\theta$$	Fully correct simplified integral with or without the 2π	A1
	$$= (2\pi)\big[-\cos\theta\big](+c)$$	Correct integration with or without the 2π	A1
	$$(2\pi)\big[-\cos\theta\big]_0^{\frac{\pi}{4}} = (2\pi)\left(-\dfrac{1}{\sqrt{2}}+1\right) \text{ Applies the limits 0 and } \dfrac{\pi}{4}.$$ Must see evidence of both limits if necessary but condone e.g. $(2\pi)\left(-\dfrac{1}{\sqrt{2}}-1\right)$ **Depends on both previous method marks.**		dM1
	TSA = $$2\pi\left(-\dfrac{1}{\sqrt{2}}+1\right) + \pi\times 1^2 + \pi\times\left(\dfrac{1}{\sqrt{2}}\right)^2$$	**Correct** expressions for the 2 "ends" and adds these to their curved surface area. **Depends on the previous method mark.**	dM1
	$$= \dfrac{\pi}{2}\left(7 - 2\sqrt{2}\right)$$	Correct answer in the required form or correct values for p and q.	A1
	Note: The final answer should follow correct work. The final mark should be withheld following e.g. $\dfrac{\mathrm{d}y}{\mathrm{d}\theta}$ clearly seen as $+\sin\theta$ or $\int \sin\theta\,\mathrm{d}\theta = +\cos\theta$ **Note:** Without the "ends" the answer is $\dfrac{\pi}{2}\left(4 - 2\sqrt{2}\right)$ **(usually scores 6/8)**		
			(8)
			Total 8

Alternative for first 4 marks:

	$$\dfrac{\mathrm{d}x}{\mathrm{d}\theta} = \dfrac{\sec\theta\tan\theta + \sec^2\theta}{\sec\theta + \tan\theta} - \cos\theta$$ Correct derivative. Do not condone missing brackets e.g. $\dfrac{\mathrm{d}x}{\mathrm{d}\theta} = \dfrac{1}{\sec\theta + \tan\theta} \times \sec\theta\tan\theta + \sec^2\theta - \cos\theta$ unless a correct expression is implied by subsequent work. Award when a correct expression is seen but note that other forms are possible e.g. $\sec\theta - \cos\theta, \ \tan\theta\sin\theta$	B1
	$$1 + \left(\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)^2 = 1 + \left(\dfrac{-\sin\theta}{\sec\theta - \cos\theta}\right)^2$$ Attempts $1 + \left(\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)^2$ with $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{\mathrm{d}y}{\mathrm{d}\theta} \times \dfrac{\mathrm{d}\theta}{\mathrm{d}x}$	M1
	$$S = (2\pi)\int \cos\theta\sqrt{1 + \left(\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)^2}\,\dfrac{\mathrm{d}x}{\mathrm{d}\theta}\,\mathrm{d}\theta$$ $$= (2\pi)\int \cos\theta\sqrt{1 + \left(\dfrac{-\sin\theta}{\sec\theta - \cos\theta}\right)^2}(\sec\theta - \cos\theta)\,\mathrm{d}\theta$$ Applies a correct surface area formula using their $\dfrac{\mathrm{d}x}{\mathrm{d}\theta}$ and their $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ with or without the 2π For reference: $\sqrt{1 + \left(\dfrac{-\sin\theta}{\sec\theta - \cos\theta}\right)^2}(\sec\theta - \cos\theta) = \tan\theta$ Allow π in front of the integral but must be an integral	M1
$(2\pi)\displaystyle\int \sin\theta \ \mathrm{d}\theta$	Fully correct simplified integral with or without the 2π	A1

Question Number	Scheme	Notes	Marks
3(a)	$y = \operatorname{arsech}\left(\dfrac{x}{2}\right) \Rightarrow \operatorname{sech} y = \dfrac{x}{2}$ $\Rightarrow \dfrac{dx}{dy} = -2\operatorname{sech} y \tanh y$	Takes "sech" of both sides and differentiates to obtain $\dfrac{dx}{dy} = k\operatorname{sech} y \tanh y$ or equivalent.	M1
	$\Rightarrow \dfrac{dx}{dy} = -2\left(\dfrac{x}{2}\right)\sqrt{1-\left(\dfrac{x}{2}\right)^2}$ M1: Replaces $\operatorname{sech} y$ with $\dfrac{x}{2}$ and $\tanh y$ with $\sqrt{1-\left(\dfrac{x}{2}\right)^2}$ A1: Correct equation involving $\dfrac{dx}{dy}$ or $\dfrac{dy}{dx}$ in any form in terms of x only.		M1A1
	$\Rightarrow \dfrac{dy}{dx} = \dfrac{-2}{x\sqrt{4-x^2}}$	Correct derivative in the required form or correct values for p and q.	A1
			(4)
(a) **Way 2**	$y = \operatorname{arsech}\left(\dfrac{x}{2}\right) \Rightarrow \operatorname{sech} y = \dfrac{x}{2}$ $\Rightarrow \cosh y = \dfrac{2}{x} \Rightarrow \sinh y \dfrac{dy}{dx} = -\dfrac{2}{x^2}$	Takes "sech" of both sides, changes to "cosh" and differentiates to obtain $\sinh y \dfrac{dy}{dx} = \dfrac{k}{x^2}$ or equivalent.	M1
	$\Rightarrow \dfrac{dy}{dx} = -\dfrac{2}{x^2 \sinh y} = -\dfrac{2}{x^2\sqrt{\left(\dfrac{2}{x}\right)^2 - 1}}$ M1: Replaces $\sinh y$ with $\sqrt{\left(\dfrac{2}{x}\right)^2 - 1}$ A1: Correct equation involving $\dfrac{dx}{dy}$ or $\dfrac{dy}{dx}$ in any form in terms of x only.		M1A1
	$\Rightarrow \dfrac{dy}{dx} = \dfrac{-2}{x\sqrt{4-x^2}}$	Correct derivative in the required form or correct values for p and q.	A1
(a) **Way 3**	$y = \operatorname{arsech}\left(\dfrac{x}{2}\right) \Rightarrow y = \operatorname{arcosh}\left(\dfrac{2}{x}\right)$ Changes to "arcosh" correctly. **Score this as the second M mark on EPEN.**		M1
	$\Rightarrow \dfrac{dy}{dx} = \dfrac{1}{\sqrt{\left(\dfrac{2}{x}\right)^2 - 1}} \times -\dfrac{2}{x^2}$ M1: Differentiates to the form $\dfrac{k}{x^2\sqrt{\left(\dfrac{2}{x}\right)^2 - 1}}$ oe A1: Correct equation involving $\dfrac{dx}{dy}$ or $\dfrac{dy}{dx}$ in any form in terms of x only. **Score this as the first M mark and first A mark on EPEN.**		M1A1
	$\Rightarrow \dfrac{dy}{dx} = \dfrac{-2}{x\sqrt{4-x^2}}$	Correct derivative in the required form or correct values for p and q.	A1

(a) Way 4	$y = \text{arsech}\left(\dfrac{x}{2}\right) \Rightarrow \text{sech}\, y = \dfrac{x}{2} \Rightarrow \left(\dfrac{x}{2}\right)^2 = \text{sech}^2\, y \Rightarrow \tanh y = \sqrt{1 - \left(\dfrac{x}{2}\right)^2}$	
	$\Rightarrow \text{sech}^2\, y \dfrac{dy}{dx} = -x\left(1 - \dfrac{x^2}{4}\right)^{-\frac{1}{2}}$ Differentiates to $\text{sech}^2\, y \dfrac{dy}{dx} = kx\left(1 - \dfrac{x^2}{4}\right)^{-\frac{1}{2}}$ or equivalent	M1
	$\Rightarrow \text{sech}^2\, y \dfrac{dy}{dx} = -x\left(1 - \dfrac{x^2}{4}\right)^{-\frac{1}{2}} \Rightarrow \dfrac{x^2}{4}\dfrac{dy}{dx} = -x\left(1 - \dfrac{x^2}{4}\right)^{-\frac{1}{2}} \Rightarrow \dfrac{dy}{dx} = -\dfrac{4}{x}\left(1 - \dfrac{x^2}{4}\right)^{-\frac{1}{2}}$ M1: Replaces $\text{sech}^2\, y$ with $\left(\dfrac{2}{x}\right)^2$ A1: Correct equation involving $\dfrac{dx}{dy}$ or $\dfrac{dy}{dx}$ in any form in terms of x only.	M1A1
	$\Rightarrow \dfrac{dy}{dx} = \dfrac{-2}{x\sqrt{4-x^2}}$ Correct derivative in the required form or correct values for p and q.	A1
(a) Way 5	$y = \text{arsech}\left(\dfrac{x}{2}\right) \Rightarrow \text{sech}\, y = \dfrac{x}{2} \Rightarrow y = \text{artanh}\left(\sqrt{1 - \left(\dfrac{x}{2}\right)^2}\right)$ Changes to "artanh" correctly. **Score this as the second M mark on EPEN.**	M1
	$\Rightarrow \dfrac{dy}{dx} = \dfrac{\frac{1}{2}\left(1 - \dfrac{x^2}{4}\right)^{-\frac{1}{2}}}{1 - \left(1 - \dfrac{x^2}{4}\right)} \times -\dfrac{x}{2}$ M1: Differentiates to the form $\dfrac{kx\left(1 - \dfrac{x^2}{4}\right)^{-\frac{1}{2}}}{1 - \left(1 - \dfrac{x^2}{4}\right)}$ oe A1: Correct equation involving $\dfrac{dx}{dy}$ or $\dfrac{dy}{dx}$ in any form in terms of x only. **Score this as the first M mark and first A mark on EPEN.**	M1A1
	$\Rightarrow \dfrac{dy}{dx} = \dfrac{-2}{x\sqrt{4-x^2}}$ Correct derivative in the required form or correct values for p and q.	A1

There may be other methods used.
If you are in any doubt if the method deserves any marks use Review.

(b)	$$f(x) = \tanh^{-1}(x) + \text{sech}^{-1}\left(\frac{x}{2}\right) \Rightarrow f'(x) = \frac{1}{1-x^2} - \frac{2}{x\sqrt{4-x^2}}$$ Correct f '(x) following through their (a) of the form $\dfrac{p}{x\sqrt{q-x^2}}$ Also allow with "made up" p and q or the letters p and q.		B1ft
	$$\frac{1}{1-x^2} - \frac{2}{x\sqrt{4-x^2}} = 0 \Rightarrow 2(1-x^2) = x\sqrt{4-x^2} \Rightarrow 4(1-x^2)^2 = x^2(4-x^2)$$ Sets $\dfrac{dy}{dx} = 0$ with their (a) of the form $\dfrac{p}{x\sqrt{q-x^2}}$ and squares both sides to reach a quartic equation		M1
	$5x^4 - 12x^2 + 4 = 0$	Correct quartic	A1
	$5x^4 - 12x^2 + 4 = 0 \Rightarrow x^2 = 2,\ 0.4$ $\Rightarrow x = ...$	Solves their quartic equation to obtain a value for x^2 and proceeds to a value for x. Apply usual rules for solving and check if necessary. Allow complex roots.	M1
	$x = \sqrt{\dfrac{2}{5}}$	Correct exact answer (allow equivalents e.g. $\dfrac{\sqrt{10}}{5}$). If any extra answers given score A0 e.g. $x = \pm\sqrt{\dfrac{2}{5}}$	A1
			(5)
			Total 9

Special case:

It is possible for a correct solution in (b) following a sign error in (a) e.g.

$$\frac{dy}{dx} = \frac{2}{x\sqrt{4-x^2}}$$

$$f(x) = \tanh^{-1}(x) + \text{sech}^{-1}\left(\frac{x}{2}\right) \Rightarrow f'(x) = \frac{1}{1-x^2} + \frac{2}{x\sqrt{4-x^2}}$$

$$\frac{1}{1-x^2} + \frac{2}{x\sqrt{4-x^2}} = 0 \Rightarrow 2(1-x^2) = -x\sqrt{4-x^2} \Rightarrow 4(1-x^2)^2 = x^2(4-x^2) \text{ etc.}$$

This is likely to score M1M1A0A0 in (a) but allow full recovery in (b) if it leads to the correct answer.

Question Number	Scheme	Notes	Marks
4(a)	$$\lambda = 3 \Rightarrow \left| \mathbf{M} - 3\mathbf{I} \right| = \begin{vmatrix} 3 & k & 2 \\ k & 2 & 0 \\ 2 & 0 & 4 \end{vmatrix} = 0 \Rightarrow 3(8) - k(4k) + 2(-4) = 0$$ or e.g. $$\left| \mathbf{M} - \lambda \mathbf{I} \right| = \begin{vmatrix} 6 - \lambda & k & 2 \\ k & 5 - \lambda & 0 \\ 2 & 0 & 7 - \lambda \end{vmatrix} = 0$$ $$\Rightarrow (6 - \lambda)(5 - \lambda)(7 - \lambda) - k(k(7 - \lambda)) + 2(0 - 2(5 - \lambda)) = 0 \Rightarrow 24 - k(4k) - 8 = 0$$ Correct interpretation of 3 being an eigenvalue leading to the formation of **a quadratic equation in k only**. If the method for forming the determinant is not clear then look for at least 2 correct "components". NB rule of Sarrus gives $24 - 8 - 4k^2 = 0$		M1
	$\Rightarrow 4k^2 = 16 \Rightarrow k = ...$	Solves quadratic. **Depends on the first M.**	dM1
	$k = \pm 2$	Correct values	A1
			(3)
(a) Way 2	$$\begin{pmatrix} 6 & k & 2 \\ k & 5 & 0 \\ 2 & 0 & 7 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = 3 \begin{pmatrix} x \\ y \\ z \end{pmatrix} \Rightarrow \begin{array}{l} 6x + ky + 2z = 3x \\ kx + 5y = 3y \\ 2x + 7z = 3z \end{array}$$ $$z = -\frac{1}{2}x, \quad y = -\frac{1}{2}kx \Rightarrow 6x - \frac{k^2 x}{2} - x = 3x \Rightarrow \frac{k^2}{2} = 2$$ Eliminates z and y and reaches a quadratic equation in k only		M1
	$\frac{k^2}{2} = 2 \Rightarrow k = ...$	Solves quadratic. **Depends on the first M.**	dM1
	$k = \pm 2$	Correct values	A1
(b)	$$k = -2 \Rightarrow \left| \mathbf{M} - \lambda \mathbf{I} \right| = \begin{vmatrix} 6 - \lambda & -2 & 2 \\ -2 & 5 - \lambda & 0 \\ 2 & 0 & 7 - \lambda \end{vmatrix}$$ $$\Rightarrow (6 - \lambda)(7 - \lambda)(5 - \lambda) + 2(2\lambda - 14) + 2(2\lambda - 10) = 0$$ Applies a value of k from (a) and a recognisable attempt at the characteristic equation (the "= 0" is not needed here). If the method is not clear then look for at least 2 correct "components".		M1
	$\Rightarrow \lambda^3 - 18\lambda^2 + 99\lambda - 162 = 0 \Rightarrow \lambda = ...$	Solves cubic. May use $\lambda = 3$ as a factor or calculator to solve. **Depends on the first mark.** Allow complex roots.	dM1
	$\lambda = 6, 9 \ (,3)$	Correct values. Allow to come from $k = 2$	A1
			(3)

(c)	$$\begin{pmatrix} 6 & -2 & 2 \\ -2 & 5 & 0 \\ 2 & 0 & 7 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = 3\begin{pmatrix} x \\ y \\ z \end{pmatrix} \Rightarrow \begin{array}{l} 6x-2y+2z=3x \\ -2x+5y=3y \\ 2x+7z=3z \end{array} \quad \Rightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} = ...$$ or $$\begin{pmatrix} 3 & -2 & 2 \\ -2 & 2 & 0 \\ 2 & 0 & 4 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \Rightarrow \begin{array}{l} 6x-2y+2z=0 \\ -2x+5y=0 \\ 2x+7z=0 \end{array} \quad \Rightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} = ...$$ Correct strategy for finding the eigenvector using a value of k from (a) Note that the cross product of any 2 rows or columns of $\mathbf{M} - 3\mathbf{I}$ gives an eigenvector		M1
	$p\begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix}$	Any correct eigenvector	A1
	$\dfrac{1}{3}\begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix}$	Any correct normalised eigenvector	A1
			(3)
			Total 9

Question Number	Scheme	Notes	Marks
5(i)	$x^2 - 3x + 5 = \left(x - \dfrac{3}{2}\right)^2 + \dfrac{11}{4}$	Correct completion of the square	B1
	$\displaystyle\int \frac{1}{\sqrt{x^2 - 3x + 5}}\, dx = \int \frac{1}{\sqrt{\left(x - \dfrac{3}{2}\right)^2 + \dfrac{11}{4}}}\, dx = \sinh^{-1}\frac{2x-3}{\sqrt{11}}(+c)$ M1: Use of $\sinh^{-1}$ A1: Fully correct expression (condone omission of $+c$) Allow equivalent correct expressions e.g. $\sinh^{-1}\dfrac{x - \frac{3}{2}}{\sqrt{\frac{11}{4}}}(+c)$, $\sinh^{-1}\dfrac{x - \frac{3}{2}}{\frac{\sqrt{11}}{2}}(+c)$ Allow equivalents for $\sinh^{-1}$ e.g. arsinh, arcsinh but **not** arsin or arcsin		M1A1
	You may see logarithmic forms for the answer: e.g. $\ln\left(\dfrac{2x-3}{\sqrt{11}} + \sqrt{\left(\dfrac{2x-3}{\sqrt{11}}\right)^2 + 1}\right)$, $\ln\left(x - \dfrac{3}{2} + \sqrt{\left(x - \dfrac{3}{2}\right)^2 + \dfrac{11}{4}}\right)$ but apply isw once a correct answer is seen.		
			(3)
(ii)	$63 + 4x - 4x^2 = -4\left(x^2 - x - \dfrac{63}{4}\right)$ $= -4\left(\left(x - \dfrac{1}{2}\right)^2 - \dfrac{64}{4}\right)$	Obtains $-4\left(\left(x - \dfrac{1}{2}\right)^2 \pm \ldots\right)$ or $-4\left(x - \dfrac{1}{2}\right)^2 \pm \ldots$ or $\ldots -(2x-1)^2$	M1
	$-4\left(\left(x - \dfrac{1}{2}\right)^2 - 16\right)$ or $64 - 4\left(x - \dfrac{1}{2}\right)^2$ or $64 - (2x-1)^2$	Correct completion of the square	A1
	$\displaystyle\int \frac{1}{\sqrt{63 + 4x - 4x^2}}\, dx = \frac{1}{2}\sin^{-1}\left(\frac{2x-1}{8}\right)(+c)$ M1: Use of $\sin^{-1}$ A1: Fully correct expression (condone omission of $+c$) Allow equivalent correct expressions e.g. $\dfrac{1}{2}\sin^{-1}\dfrac{x - \frac{1}{2}}{4}(+c)$, $-\dfrac{1}{2}\sin^{-1}\dfrac{\frac{1}{2} - x}{4}(+c)$ Allow equivalents for $\sin^{-1}$ e.g. arsin, arcsin but not arsinh or arcsinh		M1A1
			(4)
	In (ii) there are no marks for using $\displaystyle\int \frac{1}{\sqrt{63 + 4x - 4x^2}}\, dx = -\int \frac{1}{\sqrt{4x^2 - 63 - 4x}}\, dx$ But if completion of square attempted first allow M1A1 e.g. for $\displaystyle\int \frac{1}{\sqrt{63 + 4x - 4x^2}}\, dx = \int \frac{1}{\sqrt{64 - (2x-1)^2}}\, dx$ but then M0 for $= \displaystyle\int \frac{-1}{\sqrt{(2x-1)^2 - 64}}\, dx$		
			Total 7

Question Number	Scheme	Notes	Marks
6(a)	$$\int e^x \sin^n x \, dx = e^x \sin^n x - n \int e^x \sin^{n-1} x \cos x \, dx$$ Applies integration by parts to obtain $\pm e^x \sin^n x \pm \alpha \int e^x \sin^{n-1} x \cos x \, dx$		M1
	$$= e^x \sin^n x - n \left\{ e^x \sin^{n-1} x \cos x - \int e^x \left((n-1) \sin^{n-2} x \cos^2 x - \sin^n x \right) dx \right\}$$ M1: Applies integration by parts to $\pm \alpha \int e^x \sin^{n-1} x \cos x \, dx$ to obtain $$\pm e^x \sin^{n-1} x \cos x \pm \int e^x \left(\alpha \sin^{n-2} x \cos^2 x - \beta \sin^n x \right) dx$$ Or equivalent e.g. $\pm e^x \sin^{n-1} x \cos x \pm \int e^x \left(\alpha \sin^{n-2} x - \beta \sin^n x \right) dx$ (if Pythagoras applied first) A1: Fully correct expression for I_n from parts applied twice.		**d**M1A1
	$$= e^x \sin^n x - n \left\{ e^x \sin^{n-1} x \cos x - \int e^x \left((n-1) \sin^{n-2} x \left(1 - \sin^2 x\right) - \sin^n x \right) dx \right\}$$ Applies $\cos^2 x = 1 - \sin^2 x$		**d**M1
	$$= e^x \sin^n x - n \left\{ e^x \sin^{n-1} x \cos x - \int e^x \left((n-1) \sin^{n-2} x - (n-1) \sin^n x - \sin^n x \right) dx \right\}$$ $$= e^x \sin^n x - n \left\{ e^x \sin^{n-1} x \cos x - \int e^x \left((n-1) \sin^{n-2} x - n \sin^n x \right) dx \right\}$$ $$= e^x \sin^n x - n e^x \sin^{n-1} x \cos x + n(n-1) I_{n-2} - n^2 I_n \Rightarrow I_n = ...$$ Completes by introducing I_{n-2} and I_n and makes I_n the subject		**d**M1
	$$I_n = \frac{e^x \sin^{n-1} x}{n^2 + 1} (\sin x - n \cos x) + \frac{n(n-1)}{n^2 + 1} I_{n-2} \; *$$ Fully correct proof with no errors but allow e.g. the occasional missing "dx" but any clear errors must be recovered before final answer e.g. missing brackets.		A1*
			(6)

(b)		
	$$I_4 = \frac{e^x \sin^3 x}{17}\left(\sin x - 4\cos x\right) + \frac{12}{17}I_2$$ or $$I_2 = \frac{e^x \sin x}{5}\left(\sin x - 2\cos x\right) + \frac{2}{5}I_0$$ Applies the reduction formula once	M1
	$$= \frac{e^x \sin^3 x}{17}\left(\sin x - 4\cos x\right) + \frac{12}{17}\left(\frac{e^x \sin x}{5}\left(\sin x - 2\cos x\right) + \frac{2}{5}I_0\right)$$ $$= \frac{e^x \sin^3 x}{17}\left(\sin x - 4\cos x\right) + \frac{12e^x \sin x}{85}\left(\sin x - 2\cos x\right) + \frac{24}{85}e^x$$ Applies the reduction formula again and uses $I_0 = \int e^x \, dx = e^x$ to obtain an expression in terms of x	M1
	$$\int_0^{\frac{\pi}{2}} e^x \sin^4 x\, dx = \left[\frac{e^x \sin^3 x}{17}\left(\sin x - 4\cos x\right) + \frac{12e^x \sin x}{85}\left(\sin x - 2\cos x\right) + \frac{24}{85}e^x\right]_0^{\frac{\pi}{2}}$$ $$= \frac{e^{\frac{\pi}{2}}}{17} + \frac{12e^{\frac{\pi}{2}}}{85} + \frac{24e^{\frac{\pi}{2}}}{85} - \frac{24}{85}$$ Uses the limits 0 and $\dfrac{\pi}{2}$ and subtracts. **Depends on both previous marks.**	dM1
	$$= \frac{41e^{\frac{\pi}{2}}}{85} - \frac{24}{85}$$ Correct expression or correct values e.g. $A = \dots, B = \dots$	A1
		(4)
	Note that the limits may be applied as they go e.g.: M1: $\quad I_4 = \dfrac{e^{\frac{\pi}{2}}}{17}(1-0) + \dfrac{12}{17}I_2$ $I_2 = \dfrac{e^{\frac{\pi}{2}}}{5}(1-0) + \dfrac{2}{5}I_0$ $I_0 = e^{\frac{\pi}{2}} - 1$ M1M1: $I_4 = \dfrac{e^{\frac{\pi}{2}}}{17} + \dfrac{12}{17}\left(\dfrac{e^{\frac{\pi}{2}}}{5} + \dfrac{2}{5}\left(e^{\frac{\pi}{2}} - 1\right)\right)$ A1: $\quad = \dfrac{41e^{\frac{\pi}{2}}}{85} - \dfrac{24}{85}$	
		Total 10

Question Number	Scheme	Notes	Marks
7(a)	$\dfrac{x-3}{4}=\dfrac{y-5}{-2}=\dfrac{z-4}{7}\Rightarrow \mathbf{r}=\begin{pmatrix}3\\5\\4\end{pmatrix}\pm\lambda\begin{pmatrix}4\\-2\\7\end{pmatrix}$	Converts to parametric form. "$\mathbf{r}=$" is not required	M1
	$2x+4y-z=1$ $\Rightarrow 2(3+4\lambda)+4(5-2\lambda)-4-7\lambda=1$ $\Rightarrow \lambda=...(3)\Rightarrow P$ is ...	Correct strategy for finding P. Condone the use of $2x+4y-z=0$ for the plane equation.	M1
	$(15,\,-1,\,25)$	Correct coordinates. Condone if given as a vector.	A1
			(3)
(a) Way 2	$\dfrac{x-3}{4}=\dfrac{y-5}{-2}\Rightarrow x=13-2y$	Uses the Cartesian equation to find x in terms of y	M1
	$2x+4y-z=1\Rightarrow 26-4y+4y-z=1$ $\Rightarrow z=...,\ x=...,\ y=...$	Correct strategy for finding P. Condone the use of $2x+4y-z=0$ for the plane equation.	M1
	$(15,\,-1,\,25)$	Correct coordinates. Condone if given as a vector.	A1
(b)	$\begin{pmatrix}4\\-2\\7\end{pmatrix}\bullet\begin{pmatrix}2\\4\\-1\end{pmatrix}=8-8-7=-7$	Applies the scalar product between the direction of l_1 and the normal to the plane	M1
	Examples: $\phi=\cos^{-1}\dfrac{\pm7}{\sqrt{69}\sqrt{21}}=...$ $\phi=\sin^{-1}\dfrac{\pm7}{\sqrt{69}\sqrt{21}}=...$ Attempts to find a relevant angle in degrees or radians. **Depends on the first method mark.**		dM1
	$\theta=10.6°$	Allow awrt 10.6 but do **not** isw and mark the final answer. For reference $\theta=10.5965654°$	A1
			(3)
(b) Way 2	$\begin{pmatrix}4\\-2\\7\end{pmatrix}\times\begin{pmatrix}2\\4\\-1\end{pmatrix}=\begin{pmatrix}26\\-18\\-20\end{pmatrix}$	Attempts vector product of normal to Π and direction of l_1	M1
	$\sqrt{26^2+18^2+20^2}=\sqrt{21}\sqrt{69}\sin\alpha$ $\sin\alpha=\dfrac{10\sqrt{46}}{69}\Rightarrow\alpha=...$	Attempts to find a relevant angle. **Depends on the first method mark.**	dM1
	$\theta=10.6°$	Allow awrt 10.6 but do **not** isw and mark the final answer. For reference $\theta=10.5965654°$	A1

(c)	$\mathbf{a} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 4 & -1 \\ 4 & -2 & 7 \end{vmatrix} = \begin{pmatrix} 26 \\ -18 \\ -20 \end{pmatrix}$	Attempts vector product of normal to Π and direction of l_1. If no method is seen expect at least 2 correct components.	M1
	$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 13 & -9 & -10 \\ 2 & 4 & -1 \end{vmatrix} = \begin{pmatrix} 49 \\ -7 \\ 70 \end{pmatrix}$	Attempts vector product of "$\mathbf{a}$" with normal to Π to find direction of l_2	M1
		Correct direction for l_2	A1
	$\mathbf{r} = \begin{pmatrix} 15 \\ -1 \\ 25 \end{pmatrix} + \mu \begin{pmatrix} 7 \\ -1 \\ 10 \end{pmatrix}$	Depends on both previous M marks Attempts vector equation using their direction vector and their P	ddM1
		Correct equation or any equivalent correct vector equation	A1
			(5)
(c) **Way 2**	$\lambda = 1 \Rightarrow (7,\ 3,\ 11)$ lies on l_1 $\mathbf{r} = \begin{pmatrix} 7 \\ 3 \\ 11 \end{pmatrix} + t \begin{pmatrix} 2 \\ 4 \\ -1 \end{pmatrix}$ $\Rightarrow 2(7+2t)+4(3+4t)-11+t=1$ $t = -\dfrac{2}{3} \Rightarrow \left(\dfrac{17}{3}, \dfrac{1}{3}, \dfrac{35}{3} \right)$ is on l_2	Complete method to find a point on l_2	M1
	Direction of l_2 is $\begin{pmatrix} 15 \\ -1 \\ 25 \end{pmatrix} - \dfrac{1}{3}\begin{pmatrix} 17 \\ 1 \\ 35 \end{pmatrix} = \dfrac{1}{3}\begin{pmatrix} 28 \\ -4 \\ 40 \end{pmatrix}$	Uses their point and their P to find direction of l_2	M1
		Correct direction for l_2	A1
	$\mathbf{r} = \begin{pmatrix} 15 \\ -1 \\ 25 \end{pmatrix} + \mu \begin{pmatrix} 7 \\ -1 \\ 10 \end{pmatrix}$	Attempts vector equation using their direction vector and their point on l_2	ddM1
		Correct equation or any equivalent correct vector equation. Must have $\mathbf{r} =$ and not e.g. $l_2 = \dots$	A1
(c) **Way 3**	Normal to plane from l_1 $\mathbf{r} = \begin{pmatrix} 3 \\ 5 \\ 4 \end{pmatrix} + t \begin{pmatrix} 2 \\ 4 \\ -1 \end{pmatrix}$ $\Rightarrow 2(3+2t)+4(5+4t)-(4-t)=1$ $t = -1 \Rightarrow (1,\ 1,\ 5)$ is on l_2	Complete method to find a point on l_2	M1
	Direction of l_2 is $\begin{pmatrix} 15 \\ -1 \\ 25 \end{pmatrix} - \begin{pmatrix} 1 \\ 1 \\ 5 \end{pmatrix} = \begin{pmatrix} 14 \\ -2 \\ 20 \end{pmatrix}$	Uses their point and their P to find direction of l_2	M1
		Correct direction for l_2	A1
	$\mathbf{r} = \begin{pmatrix} 1 \\ 1 \\ 5 \end{pmatrix} + \mu \begin{pmatrix} 7 \\ -1 \\ 10 \end{pmatrix}$	Attempts vector equation using their direction vector and their point on l_2	ddM1
		Correct equation or any equivalent correct vector equation. Must have $\mathbf{r} =$ and not e.g. $l_2 = \dots$	A1
			Total 11

Question Number	Scheme	Notes	Marks
8(a)	$b^2 = a^2\left(1-e^2\right) \Rightarrow 4 = 9\left(1-e^2\right) \Rightarrow e = ...$ or e.g. $e = \sqrt{1 - \dfrac{b^2}{a^2}} \Rightarrow e = ...$	Uses a correct formula with a and b correctly placed to find a value for e	M1
	$e = \dfrac{\sqrt{5}}{3}$	Correct value (or equivalent) $e = \pm\dfrac{\sqrt{5}}{3}$ scores A0	A1
			(2)
(b)(i)	$(\pm ae,\ 0) = \left(\pm\sqrt{5},\ 0\right)$ or $\left(\pm 3\dfrac{\sqrt{5}}{3},\ 0\right)$ Correct foci. Must be coordinates but allow unsimplified and isw if necessary. Follow through their e so allow for $(\pm 3 \times \text{their } e,\ 0)$		B1ft
(ii)	$x = \pm\dfrac{a}{e} = \pm\dfrac{9}{\sqrt{5}}$ or $x = \pm\dfrac{3}{\frac{\sqrt{5}}{3}}$ Correct directrices. Must be equations but allow unsimplified and isw if necessary. Follow through their e so allow for $x = \pm 3/\text{their } e$		B1ft
			(2)
	Special case: Use of a^2 for a and b^2 for b **consistently** scores M0A0 in (a) and B1ft B1ft in (b) This gives $e = \dfrac{\sqrt{65}}{9}$, $\left(\pm\sqrt{65},\ 0\right)$, $x = \pm\dfrac{81}{\sqrt{65}}$		
(c)	$\dfrac{dx}{d\theta} = -3\sin\theta,\ \dfrac{dy}{d\theta} = 2\cos\theta$ or $\dfrac{2x}{9} + \dfrac{2y}{4}\dfrac{dy}{dx} = 0$ or $y = \left(4 - \dfrac{4x^2}{9}\right)^{\frac{1}{2}} \Rightarrow \dfrac{dy}{dx} = -\dfrac{4x}{9}\left(4 - \dfrac{4x^2}{9}\right)^{-\frac{1}{2}}$ $\Rightarrow \dfrac{dy}{dx} = ... \left(= \dfrac{2\cos\theta}{-3\sin\theta}\right)$	Correct strategy for the gradient of l in terms of θ. Allow $\dfrac{dy}{dx} = \dfrac{2\cos\theta}{-3\sin\theta}$ to be stated.	M1
	$y - 2\sin\theta = \dfrac{2\cos\theta}{-3\sin\theta}(x - 3\cos\theta)$	Correct straight line method (any complete method). Finding the equation of the normal is M0.	M1
	$-3y\sin\theta + 6\sin^2\theta = 2x\cos\theta - 6\cos^2\theta$ $2x\cos\theta + 3y\sin\theta = 6*$	Cso with at least one intermediate line of working	A1*
			(3)

(d)	$l_2 : \ y = \dfrac{3\sin\theta}{2\cos\theta}x$	Correct equation for l_2	B1
	$2x\cos\theta + 3y\sin\theta = 6,\ y = \dfrac{3\sin\theta}{2\cos\theta}x$ $\Rightarrow x = ...,\ y = ...$	Complete method for Q	M1
	$Q : \left(\dfrac{12\cos\theta}{4\cos^2\theta + 9\sin^2\theta},\ \dfrac{18\sin\theta}{4\cos^2\theta + 9\sin^2\theta} \right)$ Correct coordinates. Allow as $x = ...,\ y = ...$ and allow equivalent correct expressions as long as they are single fractions e.g. $x = \dfrac{12\cos\theta}{4 + 5\sin^2\theta} \quad y = \dfrac{18\sin\theta}{4 + 5\sin^2\theta}, \quad x = \dfrac{12\cos\theta}{9 - 5\cos^2\theta} \quad y = \dfrac{18\sin\theta}{9 - 5\cos^2\theta}$		A1
			(3)

(e)	At Q, $\dfrac{y}{x} = \dfrac{3}{2}\tan\theta$	Uses their coordinates of Q to attempt an equation connecting x, y and θ or states or uses the equation found in (d)	M1
	$x = \dfrac{12\cos\theta}{4\cos^2\theta + 9\sin^2\theta} = \dfrac{12\sec\theta}{4 + 9\tan^2\theta} \Rightarrow x^2 = \dfrac{144\sec^2\theta}{\left(4 + 9\tan^2\theta\right)^2} = \dfrac{144\left(1 + \dfrac{4y^2}{9x^2}\right)}{\left(4 + 9\times\dfrac{4y^2}{9x^2}\right)^2}$ or $y = \dfrac{18\sin\theta}{4\cos^2\theta + 9\sin^2\theta} = \dfrac{12\sec\theta\tan\theta}{4 + 9\tan^2\theta}$ $\Rightarrow y^2 = \dfrac{324\sec^2\theta\tan^2\theta}{\left(4 + 9\tan^2\theta\right)^2} = \dfrac{324\left(1 + \dfrac{4y^2}{9x^2}\right)\dfrac{4y^2}{9x^2}}{\left(4 + 9\times\dfrac{4y^2}{9x^2}\right)^2}$ Eliminates θ **Depends on the first mark.**		dM1
	$\Rightarrow x^2 = \dfrac{x^2\left(9x^2 + 4y^2\right)}{\left(x^2 + y^2\right)^2} \Rightarrow \left(x^2 + y^2\right)^2 = 9x^2 + 4y^2$ or $\Rightarrow 9\times 16x^2 y^2\left(1 + \dfrac{y^2}{x^2}\right)^2 = 4\times 18^2\left(1 + \dfrac{4y^2}{9x^2}\right) \Rightarrow \left(x^2 + y^2\right)^2 = 9x^2 + 4y^2$ Correct equation or correct values for α and β.		A1
			(3)
(e) **Way 2**	$x = \dfrac{12\cos\theta}{4 + 5\sin^2\theta}$ $y = \dfrac{18\sin\theta}{4 + 5\sin^2\theta} \Rightarrow \left(x^2 + y^2\right)^2 = \left(\dfrac{144\cos^2\theta + 324\sin^2\theta}{\left(4 + 5\sin^2\theta\right)^2}\right)^2$ Uses their Q to obtain an expression for $\left(x^2 + y^2\right)^2$ in terms of θ		M1
	$\left(\dfrac{144\cos^2\theta + 324\sin^2\theta}{\left(4 + 5\sin^2\theta\right)^2}\right)^2 = \left(\dfrac{144 + 180\sin^2\theta}{\left(4 + 5\sin^2\theta\right)^2}\right)^2 = \left(\dfrac{36\left(4 + 5\sin^2\theta\right)}{\left(4 + 5\sin^2\theta\right)^2}\right)^2 = \dfrac{1296}{\left(4 + 5\sin^2\theta\right)^2}$ $\dfrac{1296}{\left(4 + 5\sin^2\theta\right)^2} = \alpha x^2 + \beta y^2 = \alpha\dfrac{144\cos^2\theta}{\left(4 + 5\sin^2\theta\right)^2} + \beta\dfrac{324\sin^2\theta}{\left(4 + 5\sin^2\theta\right)^2} \Rightarrow \alpha = ..., \beta = ...$ Substitutes into the given answer and solves for α and β **Depends on the first mark.**		dM1
	$\left(x^2 + y^2\right)^2 = 9x^2 + 4y^2$	Correct expression or correct values for α and β.	A1
			Total 13